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Yaw Pointing/Lateral Translation Using Robust Sampled Data Eigenstructure Assignment

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Introduction

SOBEL and Shapiro[1] used eigenstructure assignment to design a continuous-time controller for the yaw pointing/lateral translation maneuver of the Flight Propulsion Control Coupling (FPCC) aircraft. The design of Sobel and Shapiro[1] is characterized by perfect decoupling, but the minimum of the smallest singular value of the return difference matrix at the aircraft inputs was only 0.18.

Sobel and Lallman[2] proposed a pseudocontrol strategy for reducing the dimension of the control space by using singular-value decomposition. The FPCC yaw pointing/lateral translation design of Sobel and Lallman[2] yields a minimum of the smallest singular value of the return difference matrix at aircraft inputs of 0.9835, but the lateral translation transient response has significant coupling to the heading angle.

Sobel and Shapiro[3] have proposed an extended pseudocontrol strategy. Piou and Sobel[4] extended eigenstructure assignment to linear time-invariant plants that are represented by the unified delta model,[5] which is valid both for continuous-time and sampled-data operation of the plant. Piou et al.[6] have extended Yedavalli's[7] Lyapunov approach for stability robustness of a linear time-invariant system to the unified delta system.[5] In this Note, the results of Refs. 3, 4, and 6 are used to design a robust sampled-data extended pseudocontrol eigenstructure assignment flight control law for the yaw pointing/lateral translation maneuver of the FPCC aircraft.

Problem Formulation

Consider a nominal linear time-invariant system described by (A, B, C). The corresponding sampled-data system[4] is described by (A_δ, B_δ, C) and the unified delta model[4] is described by (A_ρ, B_ρ, C). Suppose that the nominal delta system is subject to linear time-invariant uncertainties in the entries of A_ρ and B_ρ, described by $\mathrm{d}A_\rho$ and $\mathrm{d}B_\rho$, respectively. Then, the delta system with uncertainty is given by $(A_\rho + \mathrm{d}A_\rho, B_\rho + \mathrm{d}B_\rho, C)$. Here $\mathrm{d}A_\rho = \mathrm{d}A, \mathrm{d}B_\rho = \mathrm{d}B$ in continuous time and $\mathrm{d}A_\rho = \mathrm{d}A_\delta, \mathrm{d}B_\rho = \mathrm{d}B_\delta$ in discrete time. Furthermore, suppose that bounds are available on the maximum absolute values of the elements of $\mathrm{d}A$ and $\mathrm{d}B$. Define $\mathrm{d}A^+$ and $\mathrm{d}B^+$ as the matrices obtained by replacing the entries of $\mathrm{d}A$ and $\mathrm{d}B$ by their absolute values, respectively. Also, define $A_{\max}$ and $B_{\max}$ as the matrices whose entries are the element-by-element bounds on the absolute values of the entries of $\mathrm{d}A$ and $\mathrm{d}B$, respectively. Then, $\{\mathrm{d}A : \mathrm{d}A^+ \leq A_{\max}\}$ and $\{\mathrm{d}B : \mathrm{d}B^+ \leq B_{\max}\}$ where $\leq$ is applied element by element to matrices.

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Consider the constant gain output feedback control law described by $u(t) = F_\rho y(t)$ where $F_\rho = F$ in continuous time and $F_\rho = F_\delta$ in discrete time. Then, the nominal closed-loop unified delta system is given by $\rho x(t) = A_{\rho c}x(t)$ where $A_{\rho c} = A + BFC$ in continuous time and $A_\delta + B_\delta F_\delta C$ in discrete time. The uncertain closed-loop unified delta system is given by $\rho x(t) = A_{\rho c}x(t) + \mathrm{d}A_{\rho c}x(t)$ where $\mathrm{d}A_{\rho c} = \mathrm{d}A + \mathrm{d}B(FC)$ in continuous time and $\mathrm{d}A_\delta + \mathrm{d}B_\delta(F_\delta C)$ in discrete time. The reader is referred to Ref. 4 for a more detailed description.

Pseudocontrol and Robustness Results

Consider the singular-value decomposition of the matrix B_ρ given by

$$B_\rho = [U_1 \quad U_2 \quad U_0]\begin{bmatrix}\Sigma_1 & & \\ & \Sigma_2 & \\ & & 0\end{bmatrix}\begin{bmatrix}V_1^T \\ V_2^T \\ V_0^T\end{bmatrix} \tag{1}$$

where $\Sigma_1 = \mathrm{diag}[\sigma_1, \ldots, \sigma_a]$ and $\Sigma_2 = \mathrm{diag}[\sigma_{a+1}, \ldots, \sigma_b]$ and $\sigma_b \leq \sigma_{b-1} \leq \cdots \leq \sigma_{a+1} \leq \epsilon$ with ϵ small.

Lemma.[3] Let the system with the pseudocontrol $\delta(t)$ be described by

$$\rho x(t) = A_\rho x(t) + \tilde{B}_\rho \delta(t) \tag{2}$$

$$y(t) = Cx(t) \tag{3}$$

where

$$\tilde{B}_\rho = U_1 + U_2[\alpha_1, \ \alpha_2] \tag{4}$$

We design a feedback pseudocontrol for the system described by Eqs. (2–4).

Then, the true control $u(t)$ for the system described by (A_ρ, B_ρ, C) is given by

$$u(t) = \left[V_1\Sigma_1^{-1} + V_2\Sigma_2^{-1}\alpha\right]\delta(t) \tag{5}$$

Furthermore, when $\alpha = [0, 0]^T$, the control law $u(t)$ given by Eq. (5) reduces to the control law given by Eq. (20) in Ref. 2.

Theorem.[6] The system matrix $A_{\rho c} + \mathrm{d}A_{\rho c}$ is stable if

$$\sigma_{\max}\left(E_{2\max}^T P_\rho^+ E_{1\max}\right)_s < 1 \tag{6}$$

where

$$E_{1\max} = A_{\rho\max} + B_{\rho\max}(F_\rho C)^+$$

$$E_{2\max} = \left\{I_n + \Delta\left[A_\rho + B_\rho(F_\rho C)\right]\right\}^+ + (\Delta/2)E_{1\max}$$

and where P_ρ satisfies the Lyapunov equation given by

$$A_{\rho c}^T P_\rho + P_\rho A_{\rho c} + \Delta A_{\rho c}^T P_\rho A_{\rho c} = -2I_n$$

P_ρ^+ is the matrix formed by the modulus of the entries of the matrix P_ρ, and $(\cdot)_s$ denotes the symmetric part of a matrix.

Yaw Pointing/Lateral Translation Control Law Design

We consider the FPCC aircraft linearized lateral dynamics described in Ref. 2. The objective of yaw pointing control is to command the aircraft yaw heading without a change in the lateral flight path angle or bank angle.

The objective of lateral translation control is to command the lateral flight path angle without a change in yaw heading or bank angle. First, we design an eigenstructure assignment control law by using an orthogonal projection. The delta state-space matrices A_δ and B_δ are computed by using the MATLAB Delta Toolbox.[8] The sampling period Δ is chosen to be 0.02 s for illustrative purposes. The achievable eigenvectors are computed by using the orthogonal projection of the ith desired eigenvector v_i^d onto the subspace spanned by the columns of $(\gamma_i I - A_\delta)^{-1}B_\delta$. The desired eigenvalues are $\gamma_{\mathrm{dr}} = (1/\Delta)\{\exp[(-2 \pm j2)\Delta] - 1\}$ for the dutch roll mode, $\gamma_{\mathrm{roll}} = (1/\Delta)\{\exp[(-3 \pm j2)\Delta] - 1\}$ for the roll mode, and

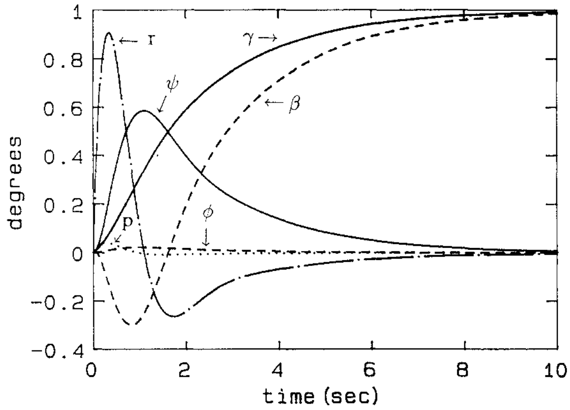

Fig. 1 Lateral translation: orthogonal projection pseudocontrol.

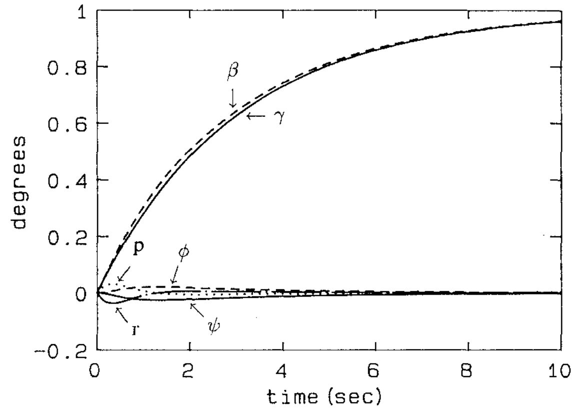

Fig. 2 Lateral translation: robust extended pseudocontrol.

$\gamma_{fp} = (1/\Delta)[\exp(-0.5\Delta) - 1]$ for the flight path mode. The desired closed-loop eigenvectors are chosen to be the same as in Sobel and Lallman.[2] Feedforward gains for steady-state tracking are computed using the command generator tracker.[9] The details of this computation for the FPCC yaw pointing/lateral translation maneuver are described in Ref. 2. The orthogonal projection solution is characterized by perfect decoupling with $\sigma_{\min}(I - FG) = 0.18$.

In order to improve the minimum singular value of $I - FG$, we design a controller by using an orthogonal projection with the pseudocontrol of Eq. (5) with $\alpha = [0, 0]^T$. The time response for lateral translation is shown in Fig. 1. This design is characterized by a lateral translation response with significant transient coupling between γ and ψ with $\sigma_{\min}(I - FG) = 0.9516$. The yaw pointing response, which is not shown, exhibits excellent decoupling and is similar to the continuous-time response shown in Ref. 2.

Next, in an attempt to obtain a robust design with excellent decoupling, we propose a new robust extended pseudocontrol design. This design method minimizes an objective function that weights the heading angle ψ due to a lateral flight path angle command γ_c and the lateral flight path angle γ due to a heading command ψ_c. Mathematically,

$$J = \sum_{k=1}^{100} \left[(1-a)\left(\psi_k^2\right)_{\gamma_c} + a\left(\gamma_k^2\right)_{\psi_c} \right] \tag{7}$$

The upper limit on the index k is chosen to include the time interval $k\Delta \in [0, 2]$ during which most of the transient response occurs. For illustrative purposes we have chosen $A_{\max} = 0.085A^+$ and $B_{\max} = 0$. After many trials, we found that a good value for the weight in Eq. (7) is $a = 0.0075$. The constraints in discrete time are 1) $|1 + \Delta\gamma| \in [e^{-4\Delta}, e^{-1.5\Delta}]$ for the time constants of the dutch roll and roll modes, 2) $|1 + \Delta\gamma| \in [\exp(-0.9\phi/[1 - (0.9)^2]^{1/2}), \exp(-0.4\phi/[1 - (0.4)^2]^{1/2})]$ for the damping ratio of the dutch roll and roll modes where $\phi = \arg(1 + \Delta\gamma)$, 3) $|1 + \Delta\gamma| \in [e^{-\Delta}, e^{-0.05\Delta}]$ for the time constant of the flight path mode, 4) $\sigma_{\max}(E_{2\max}^T P_\rho^+ E_{1\max})_s < 0.999$ for Lyapunov stability robustness, and 5) $\min_\omega \sigma_{\min}\{I - FG[(e^{j\omega\Delta} - 1)/\Delta]\} \geq 0.55$, $0 < \omega < \pi/T$, for the minimum singular value of the return difference matrix at the aircraft inputs.

The quantities that are varied by the optimization include Re γ_{dr}, Im γ_{dr}, Re γ_{roll}, Im γ_{roll}, γ_{fp}, Re $z_1(1)$, Re $z_1(2)$, Im $z_1(1)$, Im $z_1(2)$, Re $z_3(1)$, Re $z_3(2)$, Im $z_3(1)$, Im $z_3(2)$, $z_5(1)$, $z_5(2)$, and the two-dimensional pseudocontrol vector α of Eq. (5). Here, the two-dimensional complex vectors z_i contain the free eigenvector parameters. That is, the ith eigenvector v_i may be written as $v_i = L_i z_i$, where the columns of $L_i = (\gamma_i I - A_\delta)^{-1}\tilde{B}_\delta$ are a basis for the subspace in which the ith eigenvector must reside.

The optimization uses subroutine "constr" from the MATLAB Optimization Toolbox[10] and subroutine "delsim" from the MATLAB Delta Toolbox.[8] The optimization is initialized with the orthogonal projection pseudocontrol design, which yields an initial value of 20.6 for the objective function of Eq. (7) and a value of 1.66 for the right-hand side (RHS) of the robustness condition of Eq. (6). The optimization yields an optimal objective function of 0.0556, a value of 0.999 for the RHS of the robustness condition, and a value of 0.55 for the minimum of the smallest singular value of the return difference matrix at the aircraft inputs. The final eigenvalues are $\gamma_{1,2} = (1/\Delta)\{\exp[(-2.60 \pm j1.25)\Delta] - 1\}$, $\gamma_{3,4} = (1/\Delta)\{\exp[(-2.65 \pm j1.29)\Delta] - 1\}$, and $\gamma_{fp} = (1/\Delta)[\exp(-0.326\Delta) - 1]$. The feedback gain matrix, where $u = +Fy$, is

$$F = \begin{bmatrix} -0.1658 & -0.1967 & -0.0808 & 0.4895 & 0.8616 \\ 0.8390 & -1.5479 & -0.7663 & -0.8565 & -1.2679 \\ 1.1525 & -.8794 & -0.4764 & -1.9712 & -3.2578 \end{bmatrix} \tag{8}$$

The time response for lateral translation is shown in Fig. 2, where we observe the excellent decoupling achieved for the lateral translation response. The yaw pointing response, which is not shown, is similar to the excellent response obtained using the pseudocontrol approach with $\alpha = [0, 0]^T$.

Conclusions

A robust sampled-data extended pseudocontrol eigenstructure assignment flight control law was designed for the yaw pointing/lateral translation maneuver of the FPCC aircraft. Simulation results show that the new design method yields excellent time response, Lyapunov robustness to structured state-space uncertainty, and an acceptable minimum of the smallest singular value of the return difference matrix at the aircraft inputs. This design exhibits a significant improvement when compared to earlier eigenstructure assignment controllers for yaw pointing/lateral translation.

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Relation between Modified Sparse Time Domain and Eigensystem Realization Algorithm

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Introduction

A WIDELY used time-domain modal parameter estimation approach is to seek a candidate characteristic polynomial whose roots correspond to natural frequencies and damping ratios. For suboptimal solutions the polynomial may be calculated by least squares (LS) or total least squares (TLS) fitting techniques, where the latter estimates correct modal parameters statistically. The LS solutions include the Ibrahim time domain (ITD),[1] sparse time domain (STD),[2] polyreference method,[3] and eigensystem realization algorithm (ERA).[4] The TLS solution mainly utilizes a subspace orthogonality concept to separate signal and noise subspaces and then extracts signal information (including natural frequency and damping) from either subspace. In other words TLS seeks an unbiased characteristic polynomial whose roots correspond to structural modes. Polynomial root finding (or equivalently the eigenvalue problem of a companion matrix) is time consuming for a highly overspecified polynomial when noise is present in the measured data. In a realization type of solution, such as ERA, model reduction is first performed to eliminate the extraneous, computational modes and a reduced eigenvalue problem is then solved. A recently proposed modified sparse time-domain (MSTD) method (Tasker and Chopra[5]) combines the accuracy in TLS and the speed in the reduced eigenvalue problem. It has been reported to be superior to STD in both accuracy and speed. In this Note we will provide an alternative way to derive the same MSTD formulation (the second and noniterative method of Tasker and Chopra) and indicate multi-input, multi-output (MIMO) applicability. We will show by reason and example that in fact the performance of MSTD is very close to the ERA but with less bias.

New Derivation of MSTD

Consider a linear, viscously damped structure that is tested in a single-input, single-output (SISO) format. The Hankel matrix H composed of sampled impulse responses, $y_k, k = 0, 1, 2, \ldots,$ is

$$H = \begin{bmatrix} y_1 & y_2 & \cdots & \cdots & y_m \\ y_2 & y_3 & \cdots & \cdots & y_{m+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{r-1} & y_r & \cdots & \cdots & y_{m+r-2} \\ y_r & y_{r+1} & \cdots & \cdots & y_{m+r-1} \end{bmatrix}$$

$$y_k = \sum_{i=1}^{n} A_i \exp(\lambda_i k \Delta t) + \sum_{i=1}^{n} A_i^* \exp(\lambda_i^* k \Delta t)$$

where y_k contains $2n$ complex conjugate modes and λ_i contains the ith natural frequency and damping ratio.

Suppose we partition H into a submatrix and a vector in two ways:

$$H = [H_1 \quad h_1] = [h_2 \quad H_2] \tag{1}$$

where h_1 and h_2 are the last and first column vectors of H and H_1 and H_2 are their complementary submatrices. Here H_2 and H_1 are related by

$$H_2 = H_1 B \quad \text{or} \quad B = H_1^+ H_2 \tag{2}$$

where the plus superscript denotes a pseudoinverse. Matrix B has observability companion form and is upper Hessenberg. Part of the eigenvalues of the coefficient matrix B correspond to structural modes and others correspond to noise modes.

Taking the SVD of a noisy and overspecified H_1 yields

$$H_1 = P_{1s} \Sigma_{1s} Q_{1s}^T + P_{1n} \Sigma_{1n} Q_{1n}^T = H_{1s} + H_{1n} \tag{3}$$

where Σ_{1s} contains larger singular values, H_{1s} stands for the signal matrix, matrices P and Q are orthogonal, and P_{1s} and Q_{1s} span the exact left and right noiseless signal subspaces. The reduced solution is

$$B = H_{1s}^+ H_2 = Q_{1s} \Sigma_{1s}^{-1} P_{1s}^T H_2 \tag{4}$$

We further decompose H_{1s} into a product of two matrices, i.e.,

$$H_{1s} = P_{1s} \Sigma_{1s} Q_{1s}^T = S_{1s} S_{2s}^T \tag{5}$$

A smaller coefficient matrix $\bar{B}$ can be calculated by

$$\bar{B} = S_{2s}^T B (S_{2s}^T)^+ = S_{1s}^+ H_2 (S_{2s}^T)^+ \tag{6}$$

Matrices B and $\bar{B}$ are similar, or pseudosimilar, in the sense that all eigenvalues of the smaller matrix $\bar{B}$ belong to those of B. The ERA uses a balanced decomposition for S_{1s} and S_{2s} where

$$S_{1s} = P_{1s} \Sigma_{1s}^{1/2} \qquad \text{and} \qquad S_{2s}^T = \Sigma_{1s}^{1/2} Q_{1s}^T \tag{7}$$

If the reduced size of $\bar{B}$ is equal to the number of active complex conjugate modes ($= 2n$), extraneous (i.e., noise) modes will be excluded.

To demonstrate the relation between MSTD and the ERA, let

$$H_s = S_1 S_2^T = P_1 \Sigma_1 Q_1^T \tag{8}$$

The TLS solution of the polynomial vector orthogonal to H_s is

$$Q_1^T \bar{b} = 0 \qquad \text{or} \qquad S_2^T \bar{b} = 0 \tag{9}$$

Now suppose S_2 can be partitioned in two ways:

$$S_2 = \begin{bmatrix} G_1 \\ g_1^T \end{bmatrix} = \begin{bmatrix} g_2^T \\ G_2 \end{bmatrix} \tag{10}$$

where g_1^T and g_2^T are the last and first row vectors and G_1 and G_2 are their complementary submatrices, respectively. The monic characteristic polynomial vector and S_2 are orthogonal to each other, i.e.,

$$S_2^T \bar{b} = [G_1^T \quad g_1] \begin{bmatrix} b \\ 1 \end{bmatrix} = 0 \tag{11}$$

The leading coefficient, 1, may be replaced by an identity matrix if multiple inputs are used.[6] Since S_2 represents the right signal subspace of H, its submatrix G_1 also approximates the right signal subspace of H_1 (with one row shorter). Using Eq. (6), we get

$$\bar{B} = G_1^T B (G_1^T)^+ = G_1^T \left[\begin{matrix} 0 \\ I \end{matrix} \;\middle|\; -b \right] (G_1^T)^+ \tag{12}$$

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